RAMAKRISHNA MISSION VIDYAMANDIRA

(Residential Autonomous College under University of Calcutta)

SECOND YEAR

B.A./B.SC. THIRD SEMESTER (July – December) 2014

Mid-Semester Examination, September 2014

Date : 15/09/2014

MATHEMATICS (Honours)

Time : 2 pm – 4 pm

Paper : III

Full Marks : 50

[Use a separate answer book for each group]

Group – A

(*Answer **any five** questions*)

1. a) Let p, q, r & s be polynomials of degree at most 3. Which, if any, of the following two conditions is sufficient for the conclusion that, the polynomials are linearly dependent?

   i) $p(1)=q(1)=r(1)=s(1)=0$

   ii) $p(0)=q(0)=r(0)=s(0)=1$ —Justify [2]

   b) Obtain a nonsingular transformation that will reduce the quadratic form $x^2+2y^2+3z^2-2xy+4yz$ to the normal form. [3]

2. a) Let A & B be 2 matrices over the same field F such that AB is defined. Then prove that $\text{Rank}(AB)\le\min\{\text{Rank}(A),\text{Rank}(B)\}$. [3]

   b) Prove that an orthogonal set of non-null vectors in a Euclidean Space V is linearly independent. [2]

3. a) Let, $U=\left\{\begin{pmatrix}a & b\\ c & d\end{pmatrix}\in\mathbb{R}^{2\times2} \mid a=b=0\right\}$ & $W=\left\{\begin{pmatrix}a & b\\ c & d\end{pmatrix}\in\mathbb{R}^{2\times2} \mid c=d=0\right\}$ be subspaces of $\mathbb{R}^{2\times2}$.

   Show that $U\oplus W=\mathbb{R}^{2\times2}$. [2½]

   b) Let, $T:\mathbb{R}^3\to\mathbb{R}^2$ be the linear map defined by $(x_1,x_2,x_3)\mapsto(x_1+x_2,2x_3-x_1)$. Find the matrix of T with respect to standard ordered bases. [2½]

4. If a finite homogeneous system of linear equations with rational coefficients has a nontrivial complex solution, need it have a nontrivial rational solution? Give a proof or a counter example. [5]

5. If $\{\beta_1,\beta_2,...,\beta_r\}$ be an orthonormal set of vectors in a Euclidean Space V, then prove that for any vector $\alpha\in V$, $\|\alpha\|^2\ge C_1{}^2+C_2{}^2+...+C_r{}^2$, where C_i is the scalar component of α along $\beta(1\le i\le r)$. [5]

6. Let, V & W be vector spaces over a field F. Let, $\{\alpha_1,...,\alpha_n\}$ be a basis of V & $\beta_1,\beta_2,...,\beta_n$ be arbitrarily chosen (not necessarily distinct) elements of W. Then prove that, there exists a unique linear map $T:V\to W$ such that $T(\alpha_i)=\beta_i\ \forall i, 1\le i\le n$. [5]

7. a) Find all linear transformations $T:\mathbb{R}^2\to\mathbb{R}^2$ which carry the line y = x to the line y = 3x. [3]

   b) Let, W be a subspace of a vectorspace V over a field F. Let $\alpha,\beta\in V$. Then, prove that, the cosets $\alpha+W=\beta+W\Leftrightarrow(\alpha-\beta)\in W$. [2]

Group – B

8. Answer **any two** questions :

   a) The plane $ax+by+cz+d=0$ bisects an angle between a pair of planes one of which is $\ell x+my+nz=p$. Show that the equation of the other plane is

   $(a^2+b^2+c^2)(\ell x+my+nz-p)=2(a\ell+bm+cn)(ax+by+cz+d)$. [5]

b) Prove that the locus of the lines which intersect the lines $y-z=1, x=0; z-x=1, y=0$ and $x-y=1, z=0$ is $x^2+y^2+z^2-2yz-2zx-2xy=1$. [5]

c) Find the magnitude and the equations of the line of shortest distance between the lines $2x+y-z=0=x-y+2z$ & $x+2y-3z-4=0=2x-3y+4z-5$. [3+2]

9. Answer **<u>any two</u>** :

a) Two bodies of masses m and m´ are attached to the lower end of an elastic string whose upper end is fixed and hang at rest; m´ falls of; show that the distance of m from the upper end of the string at time t is $a+b+c\cos\left(\sqrt{\frac{g}{b}}t\right)$, where a is the unstretched length of the string and b is the distance by which if would be stretched when supporting m, b+c is the distance by which it would be stretched when supporting both m and m´. [7½]

b) A particle describes a rectangular hyperbola, the acceleration being directed from the centre. Show that the angle θ described about the centre in time t after leaving the vertex is given by $\tan\theta = \tanh\left(\sqrt{\mu}t\right)$ where μ is the acceleration at distance unity. [7½]

c) A particle moves in a straight line with an acceleration towards a fixed point in the straight line, which is equal to $\frac{\mu}{x^2}-\frac{\lambda}{x^3}$ when the particle is at a distance x from the given point; it starts from rest at a distance a; show that it oscillates between this distance and the distance $\frac{\lambda a}{2\mu a-\lambda}$, and that its periodic time is $\frac{2\pi\mu a^3}{(2a\mu-\lambda)^{3/2}}$. [7½]

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